Core Mathematics 4 Paper I

1. Differentiate each of the following with respect to x and simplify your answers.

(i) $\ln(\cos x)$ [2]

(ii) $x^2 \sin 3x$ [2]

2. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(ii) Find an equation for the normal to the curve at the point (3, –2). [3]

3.
$$\mathrm{f}(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2-5x-3}, \quad |x| < \tfrac{1}{2}.$$

(i) Show that

$$\mathrm{f}(x) = \frac{4x-1}{2x+1}.$$ [4]

(ii) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

4. A curve has parametric equations

$$x = t^3 + 1, \quad y = \frac{2}{t}, \quad t \neq 0.$$

(i) Find an equation for the normal to the curve at the point where $t = 1$, giving your answer in the form $y = mx + c$. [6]

(ii) Find a cartesian equation for the curve in the form $y = \mathrm{f}(x)$. [3]

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5. $$\mathrm{f}(x) = \frac{15 - 17x}{(2+x)(1-3x)^2}, \quad x \neq -2, \quad x \neq \tfrac{1}{3}.$$

(i) Find the values of the constants A, B and C such that

$$\mathrm{f}(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}.$$ [5]

(ii) Find the value of

$$\int_{-1}^{0} \mathrm{f}(x)\ \mathrm{d}x,$$

giving your answer in the form $p + \ln q$, where p and q are integers. [5]

6. Relative to a fixed origin, O, the line l has the equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ p \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ q \end{pmatrix},$$

where p and q are constants and λ is a scalar parameter.

Given that the point A with coordinates $(-5, 9, -9)$ lies on l,

(i) find the values of p and q, [3]

(ii) show that the point B with coordinates $(25, -1, 11)$ also lies on l. [2]

The point C lies on l and is such that OC is perpendicular to l.

(iii) Find the coordinates of C. [3]

(iv) Find the ratio $AC : CB$ [2]

7. *(i)* Use the substitution $x = 2\sin u$ to evaluate

$$\int_{0}^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\ \mathrm{d}x.$$ [6]

(ii) Evaluate

$$\int_{0}^{\frac{\pi}{2}} x\cos x\ \mathrm{d}x.$$ [5]

Turn over

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8. The rate of increase in the number of bacteria in a culture, N, at time t hours is proportional to N.

(i) Write down a differential equation connecting N and t. [1]

Given that initially there are N_0 bacteria present in a culture,

(ii) Show that $N = N_0 e^{kt}$, where k is a positive constant. [6]

Given also that the number of bacteria present doubles every six hours,

(iii) find the value of k, [3]

(iv) find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. [2]

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